DIRECTIONS FOR USE

of the

Automation of the calculation of Hilbert-Kunz Multiplicities and F-Signatures

Software Overview: The hilbert-kunz-and-f-sig software calculates the Hilbert-Kunz multiplicity and F-signature for intersection algebras $\mathcal{B}_R(I,J) = \bigoplus_{r,s\in\mathbb{N}}(I^r \cap J^s)$ of monomial ideals $I = (x_1^{a_1}x_2^{a_2}\cdots x_n^{a_n})$ and $J = (x_1^{b_1}x_2^{b_2}\cdots x_n^{b_n})$ in a polynomial ring $R = \mathsf{k}[x_1,\ldots,x_n]$ over a field, with $a_i, b_i \in \mathbb{N}_+$. The calculation command requires a local installation of *Mathematica*, but the inequalities command does not. **Both commands require that the integers be entered in fan order**, i.e., $a_i/b_i \geq a_{i+1}/b_{i+1}$ for all i.

One time steps

1. Download the .zip file and extract it into your chosen directory.

2. In a Bash shell, go to the chosen directory using the `cd` command(s) and run `./setup`.

Commands for regular use-in a Bash shell, in chosen directory (permuted to fan order):

`./calculate-integral` $a_1\ a_2\cdots a_n\ b_1\ b_2\cdots b_n$

`./inequalities` $a_1\ a_2\cdots a_n\ b_1\ b_2\cdots b_n$

Example 0.1. For $\mathcal{B}_R((x),(x))$, where $R = \mathsf{k}[x]$, the user enters the commands with the entries 1 1, as shown below. This intersection algebra is isomorphic to $\mathsf{k}[a,b,c,d]/(ab-cd)$.

use directory$ `./calculate-integral 1 1`

```
Hilbert-Kunz Multiplicity                  = 4/3
F-Signature                                = 2/3
```

use directory$ `./inequalities 1 1`

```
Integrate[Boole[0 <= x && 0 <= y && 1x <= z1 && 1y <= z1 &&
((z1 < 1x + 1 || z1 < 1y + 1)) &&
((y < 1 || z1 < 1x + 1)) &&
((x < 1 || z1 < 1y + 1)) &&
&& ((x < 1 || y < 1))], {x, 0, 2000}, {y, 0, 2000}, {z1, 0, 2000}]


Integrate[Boole[1x <= z1 < 1 + 1x && 1y <= z1 < 1 + 1y] &&
{x, 0, 1}, {y, 0, 1 }, {z, 0, 10}]
```

Remark 0.2. It is possible to use the program when I, J have zero exponents. For example, $\mathcal{B}_R((x),(x)) = \mathcal{B}_R(1,1)$, where each 1 represents the exponent on each x. Thus, if $S = \mathsf{k}[x_1,x_2]$ and $I = (x_1x_2), J = (x_2)$, then as per Proposition 1.6 in [1],

$$\mathcal{B}_S(I,J) = \mathcal{B}_S((x_1x_2),(x_2)) = \mathcal{B}_S((1,1),(0,1)) \cong \mathcal{B}_R(1,1)[x_1] \cong \left(\frac{\mathsf{k}[a,b,c,d]}{(ab-cd)}\right)[x_1].$$

Hence, $\mathrm{e}_{HK}(\mathcal{B}_S(I,J)) = 4/3$ and $s(\mathcal{B}_S(I,J)) = 2/3$.

Remark 0.3. The integrals being calculated are the volumes of polytopes. The details behind this automation are in the paper [2]. The mathematical theory driving the computations are in the paper [1].

References

[1] Enescu, F., Spiroff, S., *Computing the invariants of intersection algebras of principal monomial ideals*, International Journal of Algebra and Computation, **29** no. 2 (2019) 17 pages.

[2] G. Johnson and S. Spiroff, *Automating the calculation of the Hilbert-Kunz multiplicity and F-signature*, Software X, to appear.

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