The Psychometric Function: Why we should not, and need not, estimate the lapse rate.

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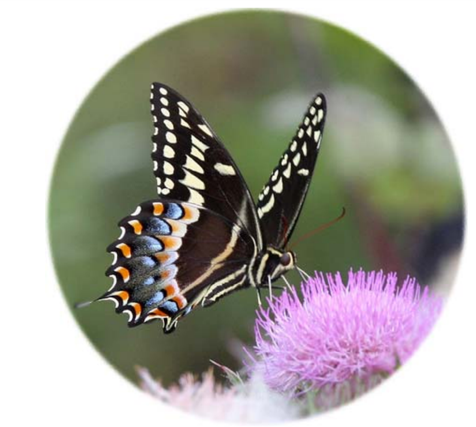
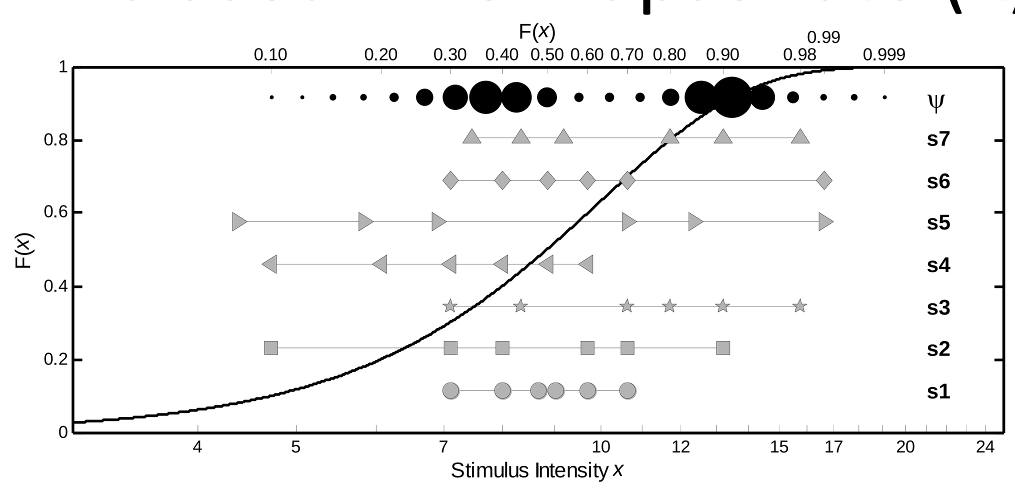


The Lapse Rate and Parameter Estimates.

Thresholds and slopes are systematically biased when lapse rate (λ) is allowed to vary.



Stimulus placement regimens used here. S1 through s7 were as in Wichmann & Hill (2001), ψ was guided by the adaptive psi method (area of filled symbols proportional to proportion of trials).

Maximum Likelihood Fits (where λ is free, it is constrained to lie within [0 0.06]):

Wichmann & Hill (2001):

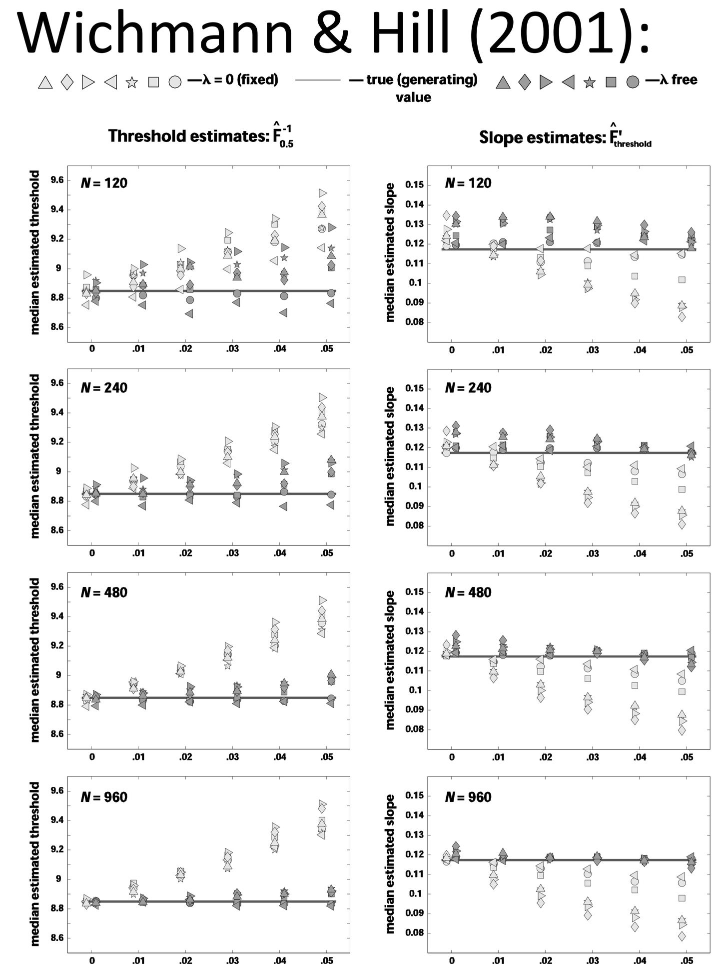
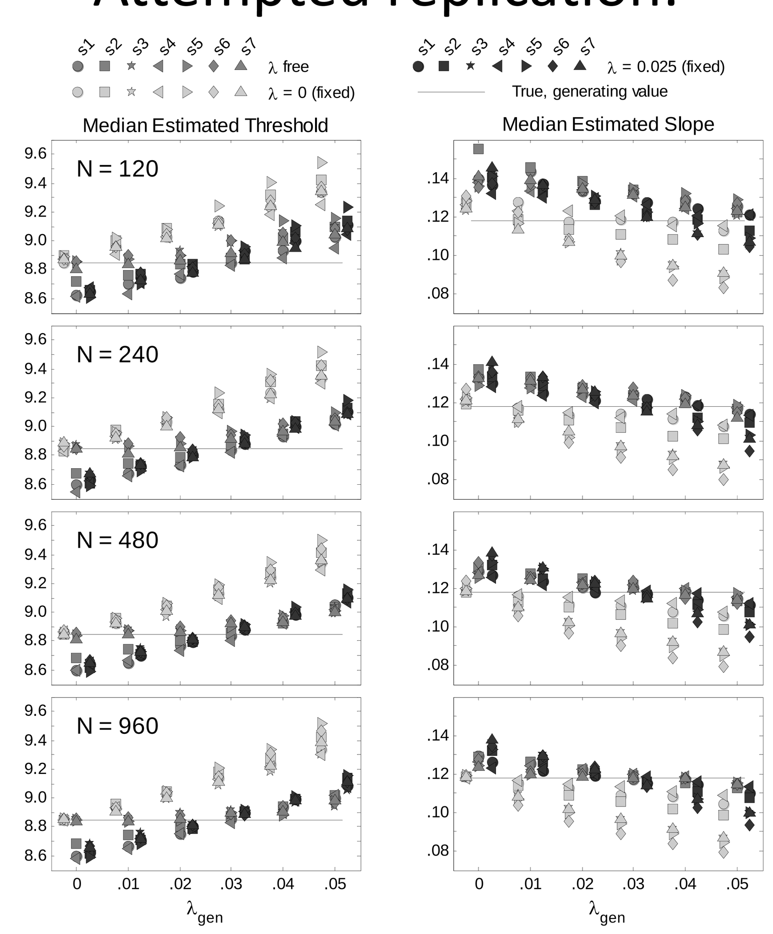


Figure 3 of Wichmann & Hill (2001). Reproduced with kind permission of Felix Wichmann.

Attempted replication:



Only when placement regimen includes high stimulus intensities AND the generating lapse rate is low are threshold estimates essentially unbiased.

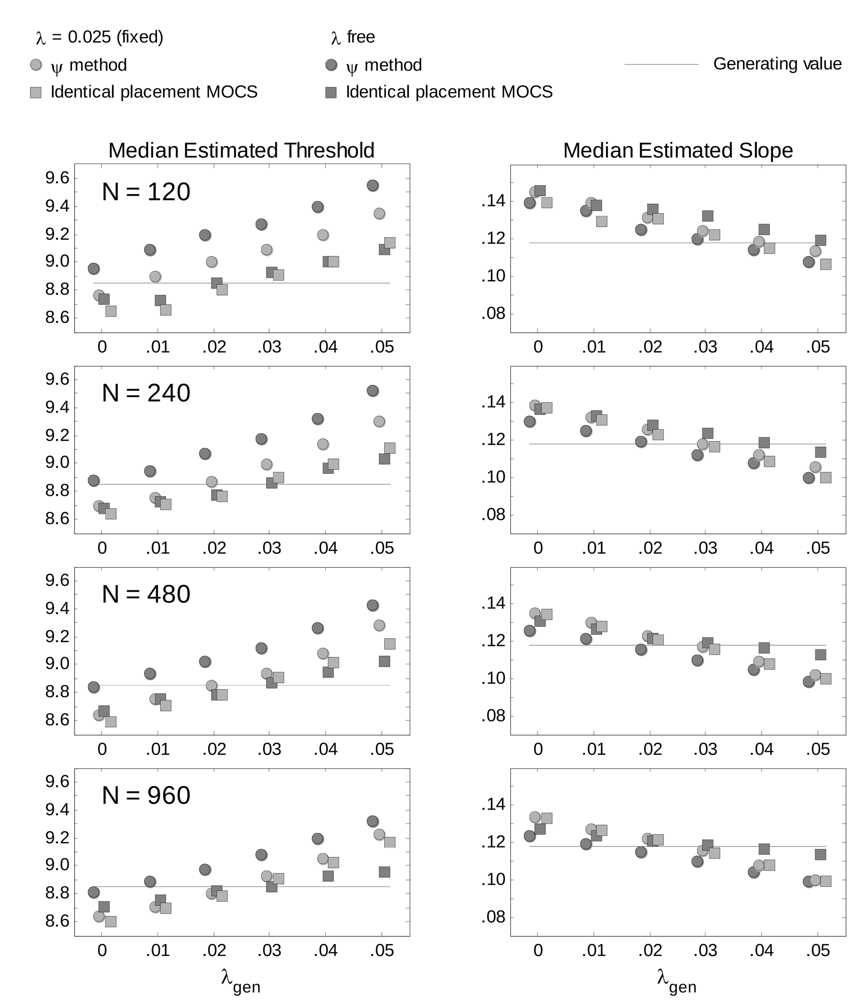
The Psychometric Function:

$$\psi(x;\alpha,\beta,\gamma,\lambda)=\gamma+(1-\gamma-\lambda)F(x;\alpha,\beta)$$

$$F_{Weibull}(x;\alpha,\beta)=1-e^{(-(x/\alpha)^{\beta})}$$

$$F_{Logistic}(x;\alpha,\beta)=\frac{1}{1+e^{(-\beta(x-\alpha))}}$$

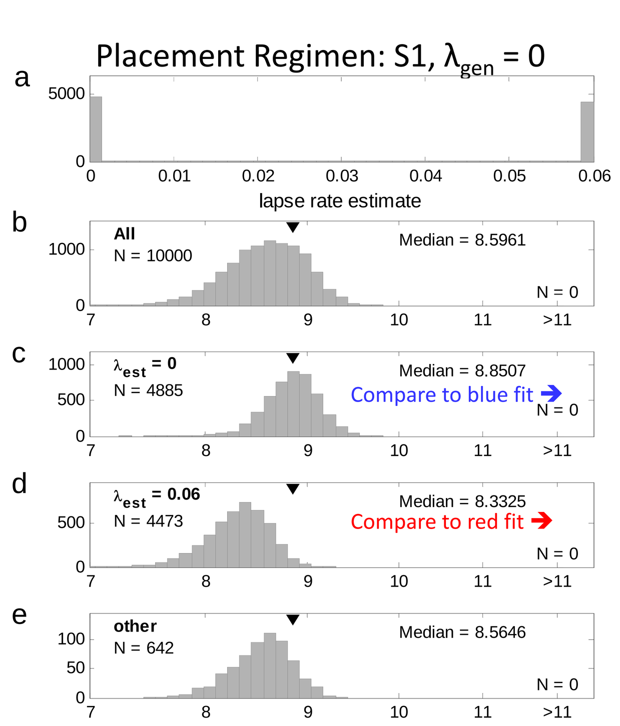
Even more so when ψ-method is used.

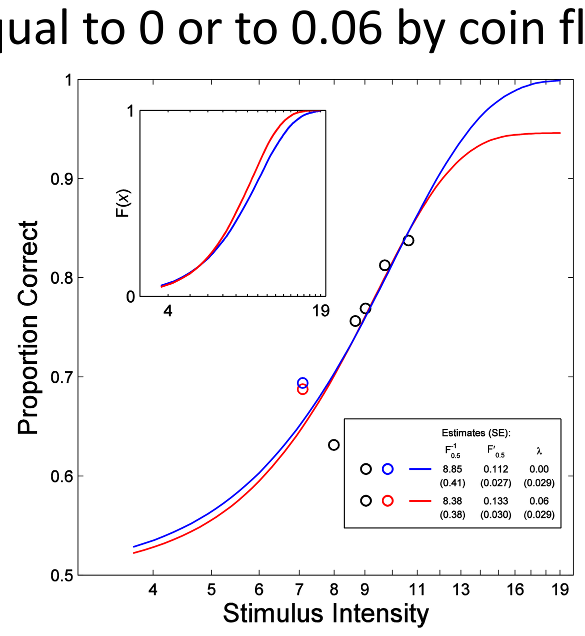


Parameter estimates when stimulus intensity is guided by ψ-method (Kontsevich & Tyler, 1999) (round symbols) or when these same intensities are retested in method of constant stimuli procedure (square symbols). Bias is influenced heavily by placement being contingent on responses (e.g., Kaernbach, 2001).

Mechanisms behind bias (simplified).

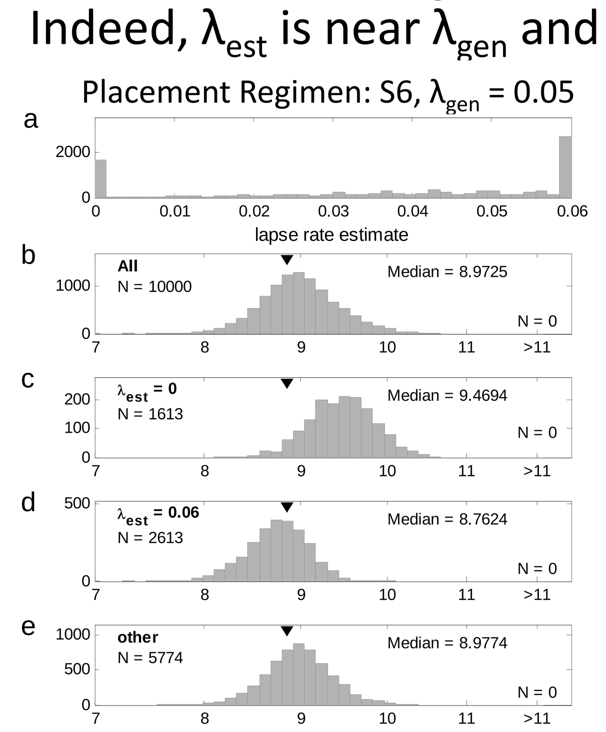
When no high intensities are included in placement regimen (s1, s2, and s4), allowing lapse rate to vary is much like assigning either a lapse rate equal to 0 or to 0.06 by coin flip:





Two identical datasets, save for the outcome of a single trial (of 960), and best fits.

In case placement regimen includes high stimulus intensity(ies) and λ_{gen} is small, there will be few incorrect responses at high intensity. One explanation only: λ_{gen} is low [and F(x) is high]. Indeed, λ_{est} is near λ_{gen} and bias in threshold is low.



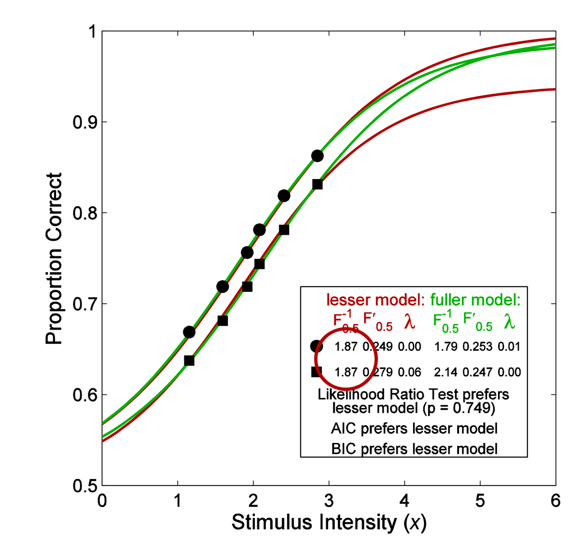
However, when λ_{gen} is high, there will be many incorrect responses at high intensity. Multiple explanations: lapse rate is high (d) or F(x) is low (c) or some combination of these two (e).

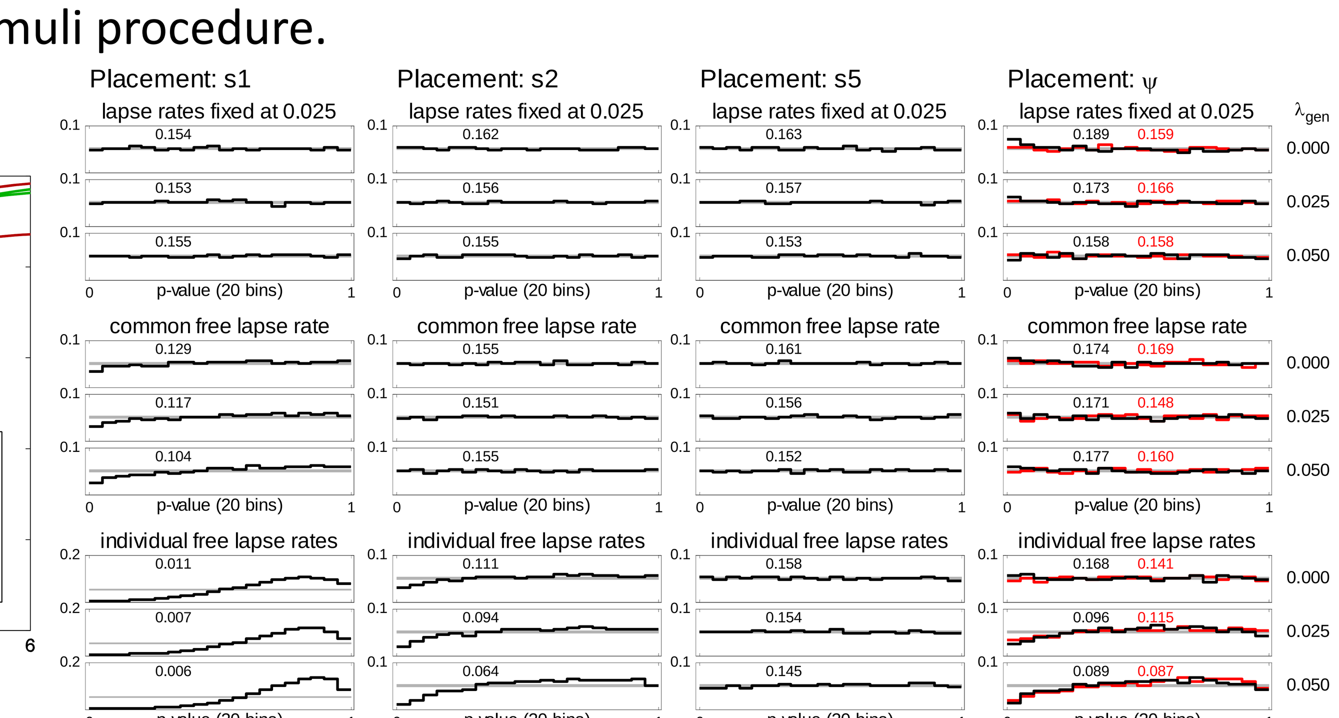
The Lapse Rate and Model Comparisons.

The Likelihood Ratio test.

Lesser model constrains thresholds between two conditions to be identical, fuller model allows separate thresholds. p-values derived from χ^2 approximation to $-2x(LL_{lesser}-LL_{fuller})$. Numbers in graphs give proportion of simulations in which fuller model was preferred by AIC. S1, s2, and s5: N = 960 (per condition), B = 10,000; ψ-method: N = 480 (per condition), B = 4,000. Gray bars: 0.05 +/- 1 (binomial) SD, red: ψ placement retested in method of constant stimuli procedure.

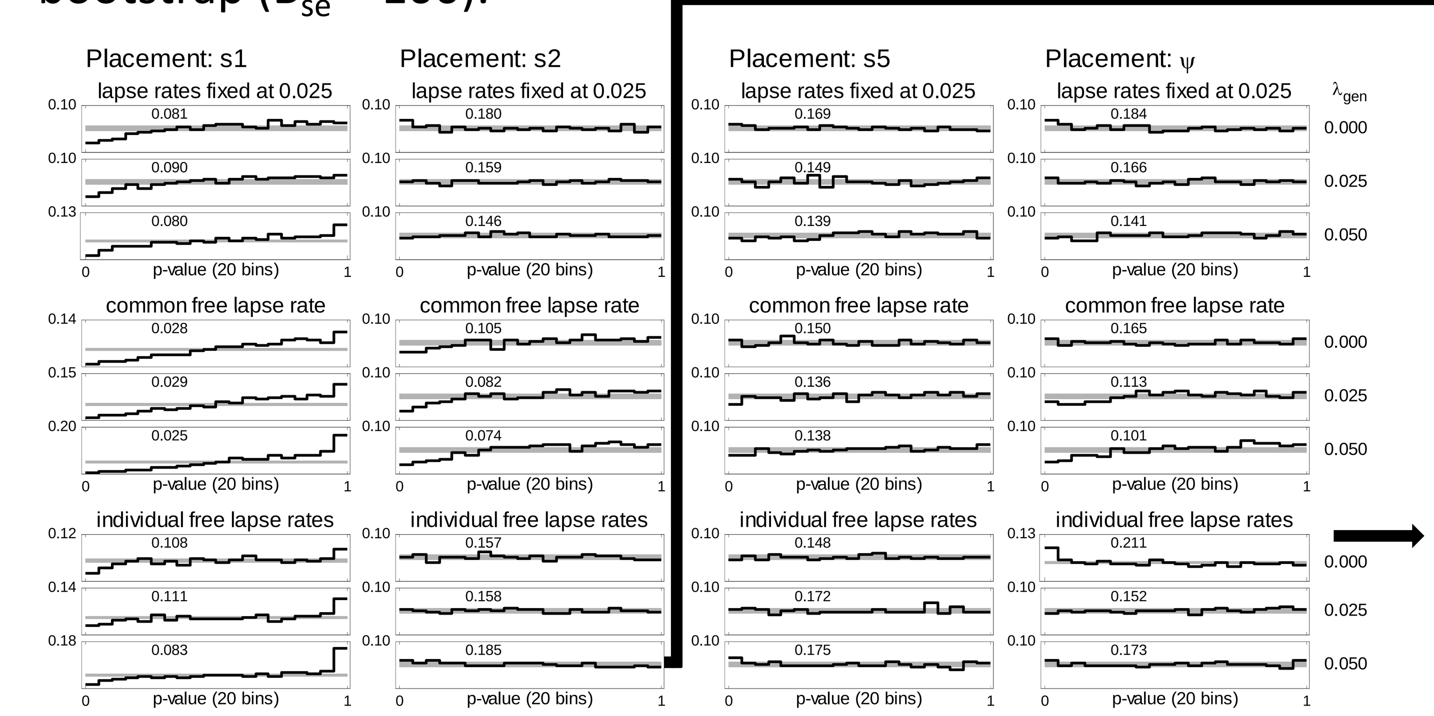
How things go wrong:

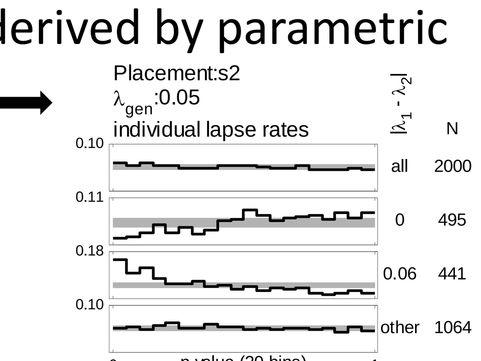


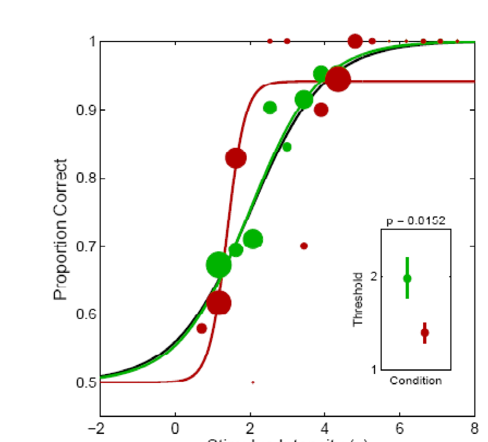


The 'eyeball' test.

In the 'eyeball' test, authors provide thresholds and SEs and leave it up to readers to make up their own mind. The oft-used rule of thumb: "If the SE bars do not overlap, I believe that thresholds are truly different" gives the numbers reported in the graphs as the proportion of simulations in which thresholds are judged truly different[1]. Since ML parameter estimates are asymptotically normally distributed, one can develop the eyeball method a bit to derive Fisherian p-values. B = 2,000. SEs derived by parametric bootstrap (B_{se} = 100).







[1]Rule of thumb would lead to 0.1573 type-I error rate if samples drawn from normal distributions with equal variance.

Main Conclusions

Freeing the lapse rate leads to modest improvement in bias over fixing the lapse rate at a small value when the method of constant stimuli is used. However, it performs much worse when an adaptive method is used and negatively affects statistical model comparisons. Statistical model comparisons are robust, however, when a fixed lapse rate which deviates from generating lapse rate is assumed.

All fits performed by Palamedes Toolbox: Prins, N. & Kingdom, F.A.A. (2009). Palamedes: Matlab routines for analyzing psychophysical data. www.palamedestoolbox.org.
Kaernbach, C. (2001). Slope bias of psychometric functions derived from adaptive data. Perception & Psychophysics, 63, 1389-1398.
Kontsevich, L.L. & Tyler, C.W. (1999). Bayesian adaptive estimation of psychometric slope and threshold. Vision Research, 39, 2729-2737.
Wichmann, F. A. & Hill, N.J. (2001). The psychometric function: I. Fitting, sampling, and goodness of fit. Perception & Psychophysics, 63, 1293-1313.