**Compressibility Factor and Pressure**  What pressure is generated when 1 lb-mol of methane is stored in a volume of 2 ft³ at 122 °F? Base calculation on each of the following:

(a) The ideal gas equation,
(b) The Redlich/Kwong equation, and,
(c) A generalized correlation.

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\[ \text{bar} := 10^5 \cdot \text{Pa} \]

\[ T := (122 + 460) \cdot R \quad V := 2 \cdot \frac{ft^3}{mol} \quad R_g := 0.7302 \cdot \frac{ft^3}{atm \cdot mol \cdot R^{-1}} \]

(a) \[ P := \frac{R_g \cdot T}{V} = \left(2.153 \cdot 10^7\right) \text{ Pa} \quad P = 212.4882 \text{ atm} \]

(b) Critical conditions and acentric factor can be obtained from Table B.1 in Appendix B:

\[ T_c := 190.6 \cdot K \quad P_c := 45.99 \cdot \text{bar} \]
\[ \omega := 0.012 \]
\[ T_r := T \cdot \frac{1}{T_c} = 1.6964 \quad P_r := P \cdot \frac{P_c}{P_c} = 4.6815 \]

Note that the reduced pressure is **high**!

\[ a := \frac{0.42748 \cdot R_g^2 \cdot T_c^2.5}{P_c} \quad b := \frac{0.08664 \cdot R_g \cdot T_c}{P_c} \]

\[ P := \frac{R_g \cdot T}{V - b} \frac{a}{T_r^{0.5} \cdot V \cdot (V + b)} = 187.6973 \text{ atm} \]

(c-1) **Generalized virial-coefficient correlation**

\[ B0 := 0.083 - \frac{0.422}{T_r^{1.8}} \quad B1 := 0.139 - \frac{0.172}{T_r^{4.2}} \]
\[ B0 = -0.0982 \quad B1 = 0.1203 \]

Calculation of the group, \( (BPc/RTc) \) in Eq. 51 of this notes:

\[ Grp := B0 + \omega \cdot B1 \quad Z := 1 + Grp \cdot \frac{P_r}{T_r} = 0.7331 \]
\[ P := \frac{Z \cdot R_g \cdot T}{V} \quad P = 155.7722 \text{ atm} \]

{n}
(c-2) Lee/Kesler generalized correlation

Now, we will have to use the linear interpolation technique in MathCad to approach the values of Z0 and Z1 in the Tables E.3 and E.4 in Smith, van Ness, and Abbott (pp.652-653, 5th ed.):

\[
\begin{bmatrix}
3.0 \\ 5.0
\end{bmatrix}
\quad
\begin{bmatrix}
1.6 \\ 1.7
\end{bmatrix}
\]

These are the points which cover the desired Pr's and Tr's in the tables.

To interpolate the functional values, Z0(Pr, Tr), we have to conduct 3 sequential interpolation steps:
1) to interpolate the Z0 at Trv=1.6,
2) to interpolate the Z0 at Trv=1.7, and,
3) to interpolate the Z0 at Trv=1.6964 (at the Tr).

**Step 1:** From Table E.1 on p. 652, we obtain the two numbers for Z0 at Trv=1.6, one corresponds to the Prv=3.0 and the other corresponds to Prv=5.0:

\[
Z01 := 
\begin{bmatrix}
0.8410 \\ 0.8617
\end{bmatrix}
\]

Then, use linear interpretation to obtain the Z0 at Pr=4.6815 (note that "linterp" is the built-in command for linear interpolation in mathCad):

\[
linterp(Prv, Z01, Pr) = 0.8584
\]

**Step 2:** Again, from the table on p.652, we obtain the two numbers for Z0 at Trv=1.7, one corresponds to the Prv=3.0 and the other corresponds to Prv=5.0:

\[
Z02 := 
\begin{bmatrix}
0.8809 \\ 0.8984
\end{bmatrix}
\]

Then, use linear interpretation to obtain the Z0 at Pr=4.6815:

\[
linterp(Prv, Z02, Pr) = 0.8956
\]

**Step 3:** Finally, we have to interpolate the two values in hand to obtain Z0 at Tr=1.6964. To achieve the goal, let us define a vector consisting the two values we obtained from the last two steps:

\[
Z0 := 
\begin{bmatrix}
linterp(Prv, Z01, Pr) \\ linterp(Prv, Z02, Pr)
\end{bmatrix}
\]

Then, use linear interpretation to obtain the Z0 at Tr=1.6964:

\[
linterp(Trv, Z0, Tr) = 0.8943
\]
To interpolate the functional values, $Z_1(Pr, Tr)$, we have to conduct 3 sequential interpolation steps similar to the procedure for $Z_0$:
1) to interpolate the $Z_1$ at $Tr_v=1.6$,
2) to interpolate the $Z_1$ at $Tr_v=1.7$, and,
3) to interpolate the $Z_1$ at $Tr_v=1.6964$ (at the $Tr$).

**Step 1:** From Table E.4 on p. 653, we obtain the two numbers for $Z_1$ at $Tr_v=1.6$, one corresponds to the $Pr_v=3.0$. and the other corresponds to $Pr_v=5.0$:

\[
Z_{I1} := \begin{bmatrix} 0.2381 \\ 0.2631 \end{bmatrix}
\]

Then, use linear interpretation to obtain the $Z_1$ at $Pr=4.6815$:

\[
\text{interp}(Pr_v, Z_{I1}, Pr) = 0.2591
\]

**Step 2:** Again, from the table on p.651, we obtain the two numbers for $Z_1$ at $Tr_v=1.7$, one corresponds to the $Pr_v=3.0$ and the other corresponds to $Pr_v=5.0$:

\[
Z_{I2} := \begin{bmatrix} 0.2305 \\ 0.2788 \end{bmatrix}
\]

Then, use linear interpretation to obtain the $Z_1$ at $Pr=4.6815$:

\[
\text{interp}(Pr_v, Z_{I2}, Pr) = 0.2711
\]

**Step 3:** Finally, we have to interpolate the two values in hand to obtain $Z_1$ at $Tr=1.6964$. To achieve the goal, let us define a vector consisting the two values we obtained from the last two steps:

\[
Z_I := \begin{bmatrix} \text{interp}(Pr_v, Z_{I1}, Pr) \\ \text{interp}(Pr_v, Z_{I2}, Pr) \end{bmatrix}
\]

Then, use linear interpretation to obtain the $Z_1$ at $Tr=1.1997$:

\[
\text{interp}(Tr_v, Z_I, Tr) = 0.2707
\]

Now, we are in a position to evaluate $Z$:

\[
Z := \text{interp}(Tr_v, Z_0, Tr) + \omega \cdot \text{interp}(Tr_v, Z_I, Tr)
\]

\[
Z = 0.8975
\]

\[
P := \frac{Z \cdot R_g \cdot T}{V} \quad P = 190.7126 \text{ atm}
\]

The experimental results shows 185 atm. Both the Lee-Kesler generalized correlation and Redlich-Kwong give reasonably good results, but the generalized virial correlation and ideal gas law completely fail at such high pressure.